Foundations of Discrete Mathematics
COT 2104

Practice 9

1) Determine which characteristics of an algorithm the following procedures have and which they lack.

   a) procedure double(n: positive integer)
      (1) while n > 0
         (a) n = 2n

   b) procedure sum(n: positive)
      (1) sum = 0
      (2) while
         (a) i < 10
         (b) sum = sum + i

2) Describe and algorithm which given, upon input of n real numbers, $a_1, a_2 \ldots a_n$ and another number, x, determines how many items in the list are equal to x.

3) Describe an algorithm which, upon input of integers a and b and a natural number n, outputs all solutions of $ax \equiv b \pmod{n}$ in the range $0 \le x \le n$, if there are solutions, and otherwise outputs the words "no solutions."

4) Solve the polynomial f(x) and each value of x using Horner's algorithm.

   a) $f(n) = -4x^3 + 6x^2 + 5x - 4$; $x = -1$
   b) $f(n) = 17x^5 - 40x^3 + 16x - 7$; $x = 3$

5) To establish a big-Oh relationship find the witnesses c and k such that $|f(x)| \le c|g(x)|$ when ever $x > k$. Determine whether each of these functions is O(x).

   a) $f(x) = 10$,   b) $f(x) = 3x + 7$, c) $f(x) = \lfloor x \rfloor$

6) Determine whether each of these functions is $O(x^2)$.

   a) $f(x) = 17x + 11$
   b) $f(x) = x^2 + 1000$
   c) $f(x) = x \log x$.

7) Show the sequence of steps in using a binary search to find the number 7 in the list 1, 2, 3, 4, 5, 6, 7, 8, 9. How many times would it be compared with an element in the list if we employed a linear search?

8) Use the definition of big-Oh given in class to show that f = O(g) in each of the following cases of functions f, g: $N \rightarrow R$

a) $f(n) = 8n^3 + 4n^2 + 5n + 1$, $g(n) = 3n^4 + 6n^2 + 8n + 2$
b) $f(n) = 17n^4 + 8n^3 + 5n^2 + 6n + 1$, $g(n) = n^4$